Engineering Tripos Part IIB FOURTH YEAR

Module 4F12: Computer Vision

Examples Paper 2

Straightforward questions are marked †
Tripos standard (but not necessarily Tripos length) questions are marked ∗

1. *Perspective projection and vanishing points*

   (a) Show that, under perspective projection, parallel lines in space meet at *vanishing points* in the image, where the locations of the vanishing points depend only on the orientations of the lines in space.

   (b) Why are the vertical lines in Renaissance paintings usually drawn without vanishing points?

   (c) What happens to parallel planes?

2. † *Rigid body transformations*

   Find the rigid body transformation matrix (specifying the mapping from world to camera coordinates) for the arrangement in Figure 1.

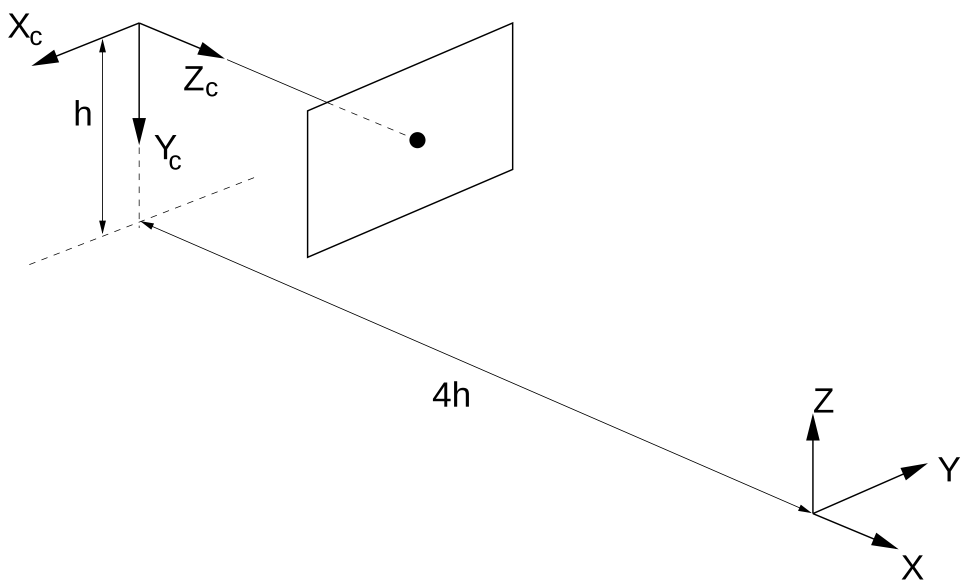

Figure 1

3. ∗ *Calibration (3D)*

   (a) Show how the overall projection matrix between a point in 3D space and its image position (in pixels) can be represented by a 3×4 matrix.

$$\begin{bmatrix} su \\ sv \\ s \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Explain the effects of the *intrinsic* and *extrinsic* camera parameters.

(b) Explain carefully how a calibration pattern of squares and a table of adjustable height can be used to calibrate a CCD camera for metrology purposes.

- Include details on the estimation of the parameters of the projection matrix by least-squares estimation.
- Explain why it is essential that the height of the table must be adjustable.
- Show how the intrinsic parameters (principal point and scale factors) and the extrinsic parameters (camera position and orientation) can be recovered from the projection matrix.

(c) Mention briefly the significance of nonlinear distortion on calibration.

4. *Planar projective transformations*

(a) Given that a unit square in the world plane

$$\mathbf{X}^p = \{(0,0),(1,0),(0,1),(1,1)\}$$

is mapped by perspective projection onto the image plane at

$$\mathbf{x} = \{(0,0),(1,-0.5),(-0.5,1),(1/3,1/3)\}$$

find the planar projective transformation that describes the mapping from $\tilde{\mathbf{X}}^p$ to $\tilde{\mathbf{x}}$ (use Matlab to solve the system of linear equations). What is the image of the point $\mathbf{X}^p = (0.5, 0.5)$?

(b) Check this answer by an appropriate geometric construction in the image plane (use graph paper).

5. ∗ *Projective transformation due to camera rotation*

(a) A camera is used to photograph a 3D scene such that the relationship between world and image pixel co-ordinates (in homogeneous co-ordinates) is given by:

$$\tilde{\mathbf{w}} = \mathbf{K}\,[\mathbf{I}|\mathbf{O}]\ \tilde{\mathbf{X}}$$

If the camera is rotated about the optical centre, show that all the image points, independent of depth, undergo a transformation given by:

$$\tilde{\mathbf{w}}' = \mathbf{K}\ \mathbf{R}\ \mathbf{K}^{-1}\ \tilde{\mathbf{w}}$$

(b) Show how the parameters of the transformation can be estimated from point correspondences in the two images, giving details of all equations and how they are solved in the case of more than 4 correspondences. Describe how to produce a *mosaic* composite image from a sequence of photographs.

6. $*$ *Line to line transformations*

   A video camera observes a train travelling along a straight section of line. The camera is calibrated by observing the image of three markers placed 0, 20 and 30m along the track. The x and y image plane coordinates of these points are $(0, 0)$, $(1, 0.5)$ and $(1.2, 0.6)$.

   Show how the train's *image* velocity can be used to determine the speed of the train on the track. Are both components of image velocity needed?

   An alarm is to warn the driver if the train's speed is greater than 40 m/s. What should be the limiting value of $\dot{y}$ (the rate of change of the train's y image plane coordinate) for the alarm to sound?

7. $*$ *Calibration (1D)*

   A video camera looks onto a table of variable height, illuminated by a projector which projects a pattern, comprising a single cross, onto the scene. The system is calibrated by adjusting the table in height to 50, 100 and 200mm. For these table positions, the centre of the cross is observed at the following image positions (in pixels):

   $$(u, v) = \{(100, 250), (140, 340), (200, 450)\}$$

   Try to determine the height of the table when the cross is observed at:

   (a) $(u, v) = (130, 310)$
   (b) $(u, v) = (170, 380)$
   (c) $(u, v) = (190, 300)$

8. $*$ *Weak perspective* (Tripos 1998)

   A *weak perspective* projection comprises an orthographic projection onto the plane $Z_c = Z_A$ followed by perspective projection onto the image plane.

   (a) Derive the homogeneous relationship between a point $(X_c, Y_c, Z_c, 1)$ and its image (su_A, sv_A, s) under weak perspective projection.

   (b) Show that the error $(u - u_A, v - v_A)$ introduced by the weak perspective approximation is given by

   $$\left((u - u_0)\frac{\Delta Z}{Z_A} \ , \ (v - v_0)\frac{\Delta Z}{Z_A} \right)$$

   where $\Delta Z \equiv Z_A - Z_c$.

   (c) Under what viewing conditions is weak perspective a good camera model? What are its advantages?

9. $*$ *Planar affine transformations*

   (a) Under what viewing conditions will the mapping from world plane to image plane be a 2D linear (affine) transformation? How many degrees of freedom does this transformation have? Explain their geometric significance.

   (b) Describe the effects of an affine transformation on parallelism, bilateral planar symmetry and centroids.

   (c) What do the additional degrees of freedom of the planar projective transformation specify?

10. $*$ *Projective invariants*

    Consider four points on a line, A,B,C and D. Under perspective projection these points project to a, b, c and d respectively. Show that the cross-ratio

    $$\frac{ac \times bd}{bc \times ad}$$

    is invariant to camera position, orientation and intrinsic parameters. By looking at permutations of the points, what other invariants can be derived? Are they independent?

Suitable past Tripos questions: 1996–2024 Q2 and Q3 (part).

Answers

1. (c) The planes $n_x X_c + n_y Y_c + n_z Z_c = d$ meet in the image at the horizon line with equation $n_x x + n_y y + f n_z = 0$.

2. $$\begin{bmatrix} X_c \\ Y_c \\ Z_c \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & h \\ 1 & 0 & 0 & 4h \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

4. (a) (0.25, 0.25)

6. (a) Only one component is needed. The alarm should sound when $\dot{y} > 2(1-y)^2$.

7. (a) 82.2mm (b) 133.1mm (c) Outlier observation.

9. (a) 6 degrees of freedom.

10. Six distinct cross-ratios can be derived. They are not independent.

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